Acta Technica 62 No. 3A/2017, 135–142

Simulation and mathematical modeling for racket position and attitude of table tennis

Jiansi Song[1]

Abstract. Racket position and attitude of table tennis robot are studied. First, relevant knowledge and method to identify racket position and its attitude is elaborated and analyzed. Then, equation-the mathematical expression, for locating racket position and attitude is set up according to table tennis movement locus. Besides, possible racket position and attitude are analyzed with practical situations. Finally, simulation platform for table tennis robot is built. Simulation experiment verified that the studied method for identifying racket position and attitude can accurately define the dropping point of the returned table tennis ball.

Key words. Table tennis robot, racket position and its attitude, mathematic modeling, movement locus, simulation platform.

1. Introduction

Analog machine for training table tennis player was simulating opponent serve and return. In order to obtain an in-depth knowledge of opponent's strategy, table tennis player should have a long-term training about a specific technical. Using table tennis robot to have those boring training can improve sports level and technological content, save labor and enhance automation level. Racket position and attitude determines whether or not it can return the ball quickly and accurately. Study on table tennis robot mainly focused on below aspects: visual measurement and control, predicting and tracking movement locus of table tennis ball and predicting hitting point [1]. However, few studies have focused on racket position and attitude. For return the ball quickly and accurately, accurately predicted movement locus and appropriate hitting point was needed because table tennis ball was moving quickly. Besides, a specific racket position and attitude was needed. Thus, method to identify racket position and attitude was studied.

[1]Shaanxi University of Science and Technology, Shaanxi, 710021, China; e-mail: jvxrhzfp@tom.com

2. Literature review

Abroad, there were many studies on table tennis robot since 1980s. In 1983, John Billincy launched a sport of robot playing table tennis [2]. In 1987, table tennis robot developed by Russel L. Anderson from American AT&T Bell Labs made man-machine competition come true [3]. Russel L. Anderson analyzed the three parts of table tennis robot thoroughly: mechanical system, real time visual system and control system. In Oct. 2012, Katharina Muelling and her team members researched and developed a table tennis robot which could learn techniques from human coach and the opponent and adjust strategies [4]. Based on those research results, various algorithms were used domestically to improve accuracy of tracking movement locus. Hitting position and speed of table tennis ball arriving at the hitting position were studied [5].

Main purpose was to identify position and attitude of racket. First, study on table tennis robot can promote the development of theories and technologies in relevant areas such as machinery, vision and simulation. Second, table tennis ball of high-speed moving was involved in table tennis robot. Besides, study on tracking rapidly moving object, predicting track and controlling mechanical arm of robot to hit ball accurately was not only critical to research on table tennis robot but also essential to fields such as military and spaceflight. Thus, there was a promising application prospect for visual tracking and track predicting of rapidly moving objects.

3. Research method

Table tennis robot that was made up of visual system, control system and mechanical system, was for table tennis training, fitting and entertainment. Visual system was used to identify table tennis ball and predict its movement locus. Control system was used to control mechanical arm according to relevant data, thus, table tennis ball would be returned at the most appropriate time and place. Mechanical system was used to control mechanical arm, which should rotate joints and move arm quickly and identify and hit table tennis ball accurately.

Below three points were involved in predicting hitting point: distance between hitting point and ground, distance between hitting point and human body in front or back, and distance between hitting point and human body on the left or right. Hitting rate and dropping point were determined by strength, speed and arc to hit [6]. In order to hit, racket position and attitude should be adjust by moving it toward the hitting point because table tennis ball was moving at high speed. Thus, to improve accuracy and effectiveness of hitting, three-dimensional coordinates of the hitting point should be predicted quickly.

Right-hand coordinate system was used for simulation platform; its coordinate system is shown in Fig. 1.

OpenGL was adopted to design simulation platform. In order to display object in the scene better, only numerical value of object was set and no units. Thus, when it was applied practically, it can be zoomed in or out correspondingly. Parameters of objects were shown as follows:

Table tennis ball: radius 0.1, original coordinate (0, 2, 8).
Table tennis table: length 16, width 8, height 3.
Racket:1 radius 0.5, thickness 0.1, original coordinate (0, 2, –8).

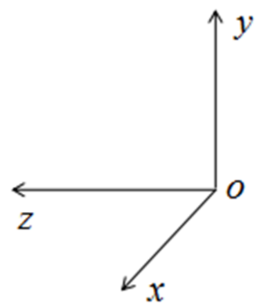

Fig. 1. Setting of coordinate system for simulation platform

There were thousands of drooping points on hall table tennis table. Table tennis robot was not as fast as players and not able to control dropping point as accurate as players, thus, a center point on the half table was set as dropping point (0, 0, 4) to determine racket position and attitude.

3.1. Determining racket position and attitude

Accurately predicted racket position and attitude and hitting point was needed to return the ball to opponent's table because table tennis ball moves quickly. Given a hitting point was known and the center point of opponent's table was the dropping point after return. Air resistance and Magnus force caused by backspin table tennis ball [7] was neglect, thus the ball was only influenced by gravity. Considering table tennis ball was moving in a two-dimensional space vertical to table and net in the middle, racket position and attitude was identified according to the known dropping point and hitting point.

3.1.1. Mathematical modeling. First, the coordinate system was set in accordance with Fig. 2.

In Fig. 2, table tennis ball was moving in plane that is parallel to plane yoz. The ball was served out from point A, reaching hitting point P after rebound from dropping point C and arrived at dropping point D after hitting. Let g denotes the gravity acceleration, H denotes the height between the point P and table tennis table, v_P denotes the ball speed at point P, β denotes included angle (the angle of incidence) between v_P and horizontal direction, S denotes the horizontal distance between point P and point D, and v_{back} denotes the speed of ball rebound from racket. Disregarding energy loss of ball rebound from racket, the formula $v_{\text{back}} = v_{\text{P}}$ was obtained. Symbol α denotes the included angle (the angle of reflection) between $_P$ and horizontal direction and T denotes the time of ball moving from point P and point D. This motion process was divided into two stages. The first stage was ball moving from point P to another point with the same height of point P and T_1 is the time spent. The second stage was ball dropping from point at the same height of point P to dropping point D and T_2 is the time spent. Thus, the below formula is satisfied.

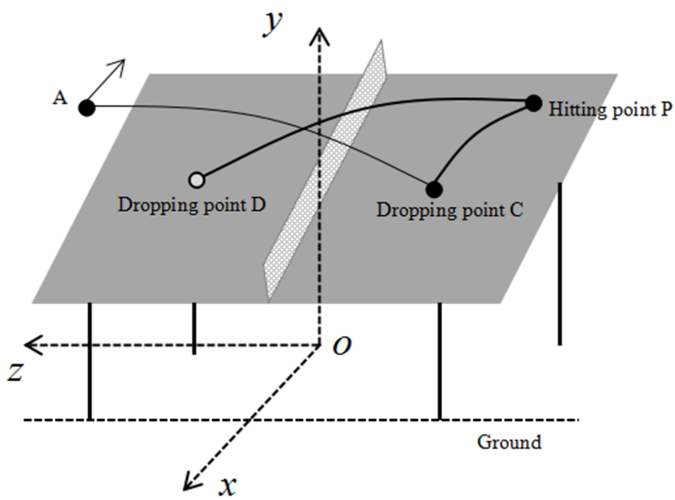

Fig. 2. Setting of coordinate system

$$S = v_{\text{back}} \cos(\alpha T), \quad 2v_{\text{back}} \sin\alpha = gT_1,$$

$$H = v_{\text{back}} \sin(\alpha T_2) + \frac{1}{2} g T_2^2, \quad T = T_1 + T_2 \,. \tag{1}$$

It can be obtained from formula (1) that

$$\frac{S^2 g^2 \tan^2\alpha}{v_{\text{back}}^2} - 2Sg\tan\alpha + \frac{S^2 g^2}{v_{\text{back}}^2} - 2gH = 0 \,. \tag{2}$$

As $\tan\alpha$ is unknown, we can determine it as follows:

$$\tan\alpha = \frac{v_{\text{back}}^2 \pm \sqrt{v_{\text{back}}^4 + 2gHv_{\text{back}}^2 - S^2 g^2}}{Sg} \,. \tag{3}$$

Put $\Delta = v_{\text{back}}^4 + 2gHv_{\text{back}}^2 - S^2 g^2$. It can be seen from formula (3) that table tennis ball can arrive at the designated dropping point D only when there was no force on racket and $\Delta \geq 0$. When $\Delta = 0$, there is only one real resolution. In this case, there is only one route that satisfies the condition and there is only one corresponding racket position and attitude. When $\Delta > 0$, there are two angles α that satisfy the condition and two corresponding racket positions and attitudes. When $\Delta < 0$, there exists no real resolution and ball cannot return to the designated dropping point. In this case, a certain amount of force should be given to hit the racket.

3.1.2. Identifying racket position and angle. It can be know from previous analysis that the prerequisite for study was the known hitting point and racket position and attitude was identified accordingly. The racket angle was identified through β and α, as can be shown from Fig. 3.

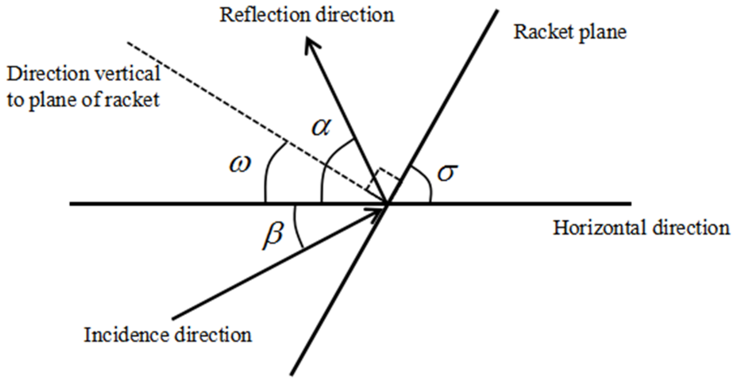

Fig. 3. Sketch map for the incidence direction and reflection direction

The figure includes acute angle σ between racket plane and horizontal direction and acute angle ω between the direction vertical to racket plane and horizontal direction. Angle $\alpha \geq \beta$ or $\alpha < \beta$, thus,

$$\omega = \frac{|\alpha - \beta|}{2}\,. \tag{4}$$

Thus,

$$\sigma = \frac{\pi}{2} - \omega = \frac{\pi}{2} - \frac{|\alpha - \beta|}{2}\,. \tag{5}$$

4. Simulation experiment and analysis of its results analysis

VC 6.0 development environment of Windows 7 operating system was adopted by the simulation platform. Single document application program was used as frame for platform and OpenGL development library was used to simulate the scene. The movement locus of table tennis ball was obtained using Runge-Kutta method [8] and shown by animation. Flow chart for simulation system is shown in Fig. 4.

After simulating the scene, program designed was operated showing the main interface of the simulation platform. For easy computing, it should be changed into front view. Then, grade of difficulty and initial parameters was set, which included initial three-dimensional coordinate of ball, initial velocity, included angle between plane of $x0y$ and $x0z$ and gravitational acceleration. Thus, simulation experiment would begin with those set initial value.

Click “hit” in the “simulation control” menu to hit the ball, thus there would be two corresponding movement locus of ball. Figures 5 and 6 are two screenshots of simulation platform for table tennis robot showing that when the designated

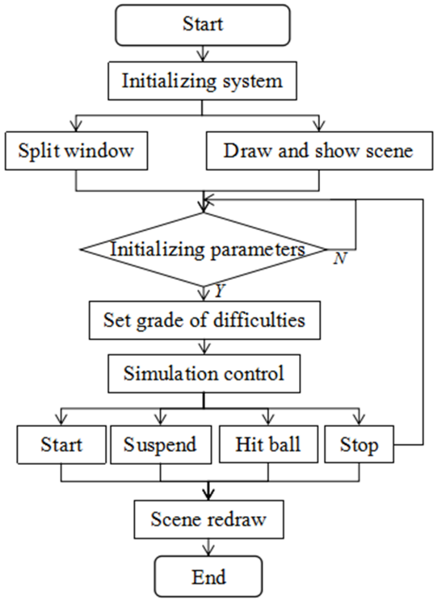

Fig. 4. Flow chart for system of simulation platform

dropping point was the same, movement locus of table tennis ball was influenced by the racket position and attitude.

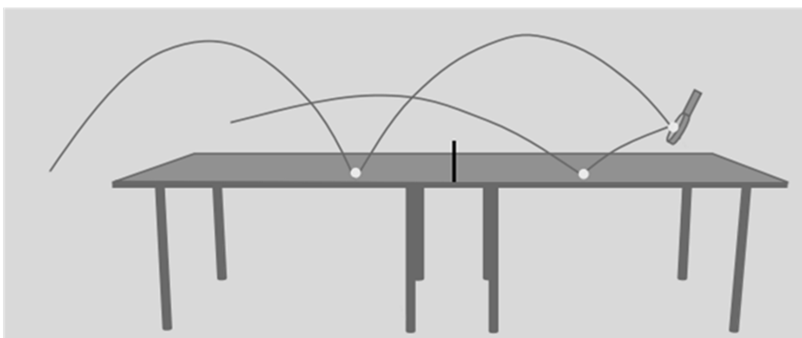

Fig. 5. Movement locus 1 for table tennis ball

In this design, it can also play, suspend, stop and draw dropping point, which is not discussed here.

5. Conclusion

Method to identify racket position and attitude of table tennis ball was studied. Mathematical modeling for racket position and attitude was built based on movement locus of table tennis ball. Thus, the method for identifying racket position

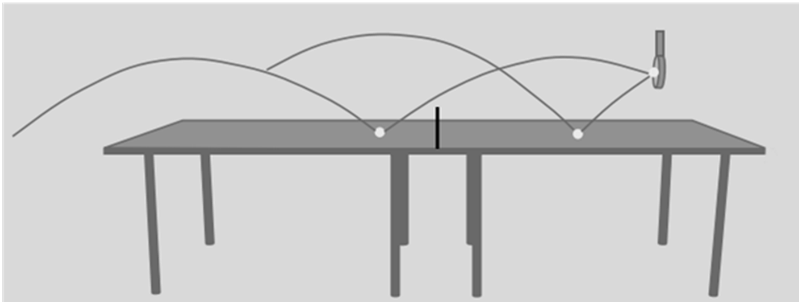

Fig. 6. Movement locus 2 for table tennis ball

and attitude was verified with simulation platform. Air resistance and Magnus force caused by backspin table tennis ball was neglected. Besides, table tennis ball movement in a two-dimensional space was studied. Therefore, three-dimensional space and those facts neglected should be included in further study.

References

[1] J. S. Tomar, D. C. Gupta, N. C. Jain: *Free vibrations of an isotropic non-homogeneous infinite plate of parabolically varying thickness*. Indian Journal of Pure Applied Mathematics *15* (1984), No. 2, 211–220.

[2] J. S. Tomar, A. K. Gupta: *Effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions*. J Sound and Vibration *98* (1985), No. 2, 257–262.

[3] R. H. Gutierrez, P. A. A. Laura: *Fundamental frequency of vibrating rectangular, non-homogeneous plates*. Applied Acoustics *18* (1985), No. 3, 171–180.

[4] R. P. Singh, S. K. Jain: *Free asymmetric transverse vibration of parabolically varying thickness polar orthotropic annular plate with flexible edge conditions*. Tamkang Journal of Science and Engineering *7* (2004), No. 1, 41–52.

[5] M. N. Gaikwad, K. C. Deshmukh: *Thermal deflection of an inverse thermoelastic problem in a thin isotropic circular plate*. Applied Mathematical Modelling *29* (2005), No. 9, 797–804.

[6] S. Chakraverty, R. Jindal, V. K. Agarwal: *Flexural vibrations of non-homogeneous elliptic plates*. Indian Journal of Engineering and Materials Sciences *12* (2005) 521–528.

[7] N. L. Khobragade, K. C. Deshmukh: *Thermal deformation in a thin circular plate due to a partially distributed heat supply*. Sadhana *30* (2005), No. 4, 555–563.

[8] Y. F. Zhou, Z. M. Wang: *Vibrations of axially moving viscoelastic plate with parabolically varying thickness*. J Sound and Vibration *316* (2008), Nos. 1–5, 198–210.

Received May 7, 2017